Nanophotonic Slow-Light Structure For Telecommunication Applications

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Abstract. In this thesis, the delay performance of slow light optical pulses inside PCWs is considered in the linear and nonlinear propagation regime from both a theoretical and an application point of view. It is numerically shown that for rates of $40Gb/s$ and $100Gb/s$, nonlinear solitary pulses experience less broadening than the linear case and can therefore be used to obtain larger delays. The storage capacity of slow light PCWs is maximized using a systematic procedure based on the optimization of various parameters of the structure. Moreover, approximate analytical expressions for the estimation of the degenerate four-wave mixing (FWM) conversion efficiency in slow-light PCWs are presented. The derived formulas incorporate the different effective modal areas and the frequency-dependent linear and nonlinear parameters of the pump, signal, and idler waves. The influence of linear loss, two-photon absorption, and free-carrier generation is also accounted for. We discuss the optimization of PCWs for FWM applications, taking into account linear loss and free-carrier effects. Suitable figures of merit are introduced in order to guide us through the choice of practical, high-efficiency designs requiring relatively low pump power and small waveguide length. Promising waveguide designs are identified, altering some structural parameters. These designs are identified using an optimization process taking into account sophisticated figure-of merits that depend on the pump bandwidth and the signal/pump tunability. We also present alternative designs that are less efficient but have smaller power requirements and are far more compact.

Keywords: Photonic crystal waveguides, Slow-light, Four-wave mixing, Soliton, Delay Lines.

1 Introduction

Photonic crystals are formed by periodically modulating the refractive index of the material in all three directions. Such structures are known to prevent light from propagating in certain directions with specified frequencies, an ability usually referred to as photonic band gap. Researchers devote a considerable amount

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of attention to photonic band gaps and with good reason. Many of the promising applications of two- and three- dimensional photonic crystals to date hinge on the location and width of photonic band gaps. For example, a crystal with a band gap might make a very good, narrow-band filter, by rejecting all frequencies in the gap. A resonant cavity, carved out of a photonic crystal, would have perfectly reflecting walls for frequencies in the gap. The simplest possible photonic crystal consists of alternating layers of material with different dielectric constants. A two-dimensional photonic crystal is periodic along two of its axes and homogeneous along the third axis. Usually a two dimensional photonic crystal is formed by embedding holes of a low refractive index material in a triangular lattice, to a higher refractive index material. Another way is to form a square lattice of dielectric columns in a lower dielectric environment. The latter is a less attractive option because such photonic crystals experience a narrower band gap as well as increased linear losses due to light scattering. In a practical application, light must be confined in all three dimensions, necessitating the usage of a three dimensional photonic crystal, which is a dielectric structure with periodicity along three different axes. However, a three dimensional photonic crystal has certain weaknesses in both fabrication and practical application. In practice it is more common to combine band gap with index guiding, creating photonic crystal slabs. A photonic crystal slab is a hybrid structure formed by adopting a two dimensional photonic crystal structure and confine light in the third dimension through means of internal reflection. For example, as a photonic crystal slab can be considered a silicon membrane embedded with holes of air in a triangular lattice. In this case, light will be confined in the third direction by layers of a lower dielectric material above and below the slab. In case these layers are filled with air, the photonic crystal is called air membrane PCW.

Introducing a defect in the photonic crystal, (i.e. by removing a line of holes along the propagation direction of a photonic crystal slab), a defect mode (or guided mode) appears inside the photonic band-gap. The localization of the waveguide mode relies on both the band gap within the plane of periodicity and also on index guiding in the vertical direction. One of the remarkable properties of this mode is that at a given frequency range, propagation occurs with an increased group index $n_g = c/|v_g|$, where v_g is the group velocity (which may be positive or negative depending on the slope of the dispersion curve) and c is the speed of light in vacuum. This phenomenon is widely known as slow-light.

2 Slow-Light

In this dissertation an extended description of the slow-light effect is presented. In general, slow light occurs due to large first order dispersion $dk/d\omega$ arising from the resonance of light with a material or structure, where k is the wave number and ω is the angular frequency. The most noticeable method that uses material dispersion in order to manipulate light is the electromagnetically induced transparency (EIT). This method holds the slow-light record at 17m/s using Bose-Einstein Condensates (BEC). The extremely low v_g in EIT only allows a bandwidth of the order of kHz. For structural dispersion methods, Δn is defined not for a material index but for an equivalent index of mode distributing over multiple materials that form the structure. Therefore, structural dispersion methods have similar problems in terms of the bandwidth and dispersion, although they are suitable for room temperature on-chip applications. Photonic crystals falls into the category of structural dispersion, forming standing waves on the Bragg condition of their periodic structure (usually called band edge) and slow light occurs due to large first order dispersion near the Bragg condition. Presently, it is straightforward to observe experimentally a group velocity of $c/10$ $c/100$ and a delay of 10ps order.

3 PlaneWave Expansion Mode Solver

This thesis reviews the most important numerical techniques for the solution of partial differential equations that can be applied to obtain the band diagram, transmission spectra and field patterns of the photonic crystal slab waveguide (PCSW). Each numerical method has each own particular strengths and weaknesses. As thoroughly explained in this thesis, these numerical methods are solving the eigenvalue problem in time or frequency domain. In our calculations, we have implemented a three dimensional plane wave expansion method mode solver based on the minimization of the Rayleigh quotient (usign the Rayleigh-Ritz method), which we have implemented in MATLAB. On a computer, this eigenequation must be discretized into N degrees of freedom using the planewave expansion method. In general, such a discretization yields a finite generalized eigen-problem $\mathbf{A}x = \omega^2\mathbf{B}x$, where $\mathbf{A}$ and $\mathbf{B}$ are N matrices and x is the eigenvector. Since the original eigen-problem is Hermitian, the discretization can be chosen so that $\mathbf{A}$ and $\mathbf{B}$ are Hermitian and $\mathbf{B}$ is positive-definite. This realization leads to iterative methods, which compute a small number p of the eigenvalues and eigenvectors, such as the p smallest eigenvalues. There are many such methods, but they share a few critical features. Firstly, they work by taking a starting guess for x (e.g., random numbers) and applying some process to iteratively improve the guess, converging quickly to the true eigenvector. In this way, any desired accuracy can be obtained in a small number of steps. Secondly, they merely require you to supply a fast way to compute the matrix-vector products $\mathbf{A}x$ and $\mathbf{B}x$. The dimensions of the supercell must be chosen carefully for two main reasons. Firstly, must be large enough to contain the photonic crystal defect that forms the waveguide. In addition, the supercell size must prevent coupling with unwanted waveguides, formed by the periodic repositioning of the supercell. Moreover, near a dielectric interface one must average the dielectric in two different ways according to effective-medium theory, depending upon the polarization of the incident light relative to the surface normal $\hat{n}$. As stated in this thesis, not doing so can lead to suboptimal convergence of the frequencies as a function of N, due to the problems of representing discontinuities in a Fourier basis. It has been shown, that using a smoothed, effective dielectric tensor near dielectric interfaces can circumvent these problems, and achieve accurate results for moderate N. The desired modes (guided modes) of the photonic crystal waveguide lie in a known frequency range (the band gap) in the interior of the spectrum. Ideally, one would like to compute only the defect modes in the band gap, without waste computation and memory on finding all the folded modes below them. In this thesis, a technique for calculating only the guided mode located inside the photonic band gap is unfolded. Applying this technique one will see that despite the fact that much fewer eigenvalues need to be found, the convergence is slower than before due to lack of appropriate preconditioning. The fact that a more suitable precondintioner needs to be constructed makes the interior eigenvalue method the less efficient choice.

4 Major Limitations of Photonic Crystals

This thesis contributes to the debate and understanding of the propagation factors limiting the applicability of photonic crystal waveguides. Slow-light in photonic crystals tends to coincide with high dispersion, which removes most of the advantages of operating in the slow light regime and severely limits the bandwidth that can be utilized. Moreover, linear losses are another issue currently being debated. It has been proposed that losses in photonic crystal waveguides scale as the square of the slowdown factor, $S = c/v_g$. The dispersion effects as well as the linear loss level is modelled and examined for PCSW. It is shown that dispersion effects are not an intrinsic property of the structure, but subject to design. Designs based on a better understanding of slow light operation can overcome this limitation, as already shown by several authors. Propagation in the aforementioned slow light regime can find use in a variety of practical applications including optical delay lines and enhanced lightmatter interaction. Given the dispersion relation of the mode, the coefficients β_2 and β_3 can be easily extracted using polynomial fitting on the dispersion relation. The group velocity dispersion (GVD) coefficient, β_2 is usually larger in the slow light regime, potentially leading to significant pulse broadening, especially at high data rates. To some extent, careful waveguide design can be applied in order to reduce the propagation losses while at the same time obtain lower values of β_2. The amount of pulse broadening can be quantified in terms of the broadening factor BF, which is defined by the ratio,

$$BF = \sigma(L)/\sigma(0) \tag{1}$$

,where $\sigma(x)$ is the root-mean-square (RMS) pulsewidth. This thesis presents a BF study for several photonic crystal waveguide designs. The presented results suggests that GVD can cause a severe amount of broadening increasing for higher bit rates and that TOD induced broadening is significantly less important. This provides a first motivation for considering soliton pulses since, SPM can compensate for GVD-induced broadening. The soliton pulse shape depends on the sign of the GVD coefficient β_2. If β_2 is positive (which is the case in the majority of the photonic crystal waveguides considered in this thesis), then dark solitons should be launched in the structure. The input soliton pulse-width T_0 is related to the corresponding full-width at half maximum duration, T_{FWHM}, of the pulse through $T_0 = T_{FWHM} < 1.76$. The pulse peak power P_0 is determined by the following expression, $P_0 = |\beta_2|/\gamma T_0^2$. The presented results suggest that for the W1 waveguide, P_0 is initially increasing with n_g and then saturates near $n_g = 15$. For $n_g > 15$, P_0 slowly decreases. This behavior is a consequence of the interplay between β_2 and , which in the case of the W1 waveguide are both increasing monotonically with n_g.In the nonlinear regime, the residual pulse broadening due to the propagation losses and TOD can be studied by numerically solving the propagation equation using the split step Fourier (SSF) method. In order to simulate the propagation of dark solitons, measures should be taken to prevent the truncation of the bright background of the pulse at the edges of the SSF time window. The broadening factor can be calculated after removing the white background from the output waveform. The presented results show that for data rates of interest in optical networking (such as 40 Gb/s and 100 Gb/s), soliton pulses can be used to obtain significant improvement in terms of the achievable delay and broadening level compared to linear pulses. We also provide a comparison between the broadening factors obtained in the linear and the nonlinear regime for data rates of 40Gb/s and 100Gb/s. It is shown that for all waveguide designs, nonlinear propagation can be quite beneficial, especially at 100Gb/s, increasing the delay obtained from each particular waveguide at a given broadening factor. We also discuss the effect of varying loss level and the benefit of launching solitons at higher peak power. Chapter 4 also highlights the relation between the propagation losses, the achievable delay, and the amount of pulse broadening experienced in a photonic crystal slab waveguide in both the linear and the nonlinear regime. The propagation loss coefficient Γ is given by the expression,

$$\Gamma = c_1 \rho_{OP} n_g + c_2 \rho_{BS} n_g^2 \tag{2}$$

This expression corresponds to the total propagation losses that encompass the intrinsic loss, the disorder-induced scattering and the losses due to out-of-plane propagations losses. The coefficients c_1 and c_2 can be extracted from measurements and depend on the fabrication method, the dielectric contrast Δ_ε, and the disorder parameter σ_d. The scattering coefficients ρ_{BS} and ρ_{OP} encompass the influence of the mode shape. They correspond to backscattering and out-of-plane scattering, respectively. In the waveguide designs under consideration, we have found that out-of-plane scattering has only marginal influence and hence can be ignored. Assuming that $\rho_{OP} \cong 0$, it is easy to relate the losses of any PCSW to those of a standard W1 waveguide fabricated with the same index contrast and disorder parameter, i.e.,

$$\Gamma(n_g) = \Gamma'(n_{g0})(\frac{n_g}{n_{g0}})^2 \frac{\rho_{BS}(n_g)}{\rho_{BS}(n_{g0})} \tag{3}$$

where $\Gamma(n_{g0})$ is the loss coefficient of the W1 waveguide calculated at $n_g = n_{g0}$ and $\rho'_{BS}(n_{g0})$ is the corresponding backscattering coefficient of the W1 waveguide. This dissertation contributes also to the impact of multiple scattering in the propagation of the pulse. Multiple scattering is the process by which the backscattered light is coupled back to the forward propagating mode and manages to reach the output. Multiple scattering manifests as a number of random peaks in the normalized transmission spectra of the waveguide. The measured linewidth of the fluctuations of the transmission of a photonic crystal waveguide is estimated to be about 10 GHz for $\lambda = 1550nm$. To illustrate the impact of these fluctuations, we consider the spectrum of a Gaussian pulse filtered by a randomly fluctuating transmission curve,

$$H(f) = \sum H_m e^{j2\pi mf/\Delta f}, \tag{4}$$

where Δf is the spectrum of the pulse and the index m runs as $1mN_p$. $H(f)$ corresponds to the transmission curve of a waveguide segment comparable to the localization length. We choose $N_p = 16$, large enough in order to produce 10-GHz-spaced fluctuations. From the presented results it is deduced that the main pulse shape remains practically unchanged but a small part of the initial energy gives rise to nearby trailing small amplitude pulses (as expected in the case of a multiply backscattered signal). However, the pulse peak power and broadening factor remain practically unchanged, suggesting that propagation is not severely affected by this effect. The above considerations suggest that these narrow transmission fluctuations are not expected to severely impact the propagation of high bandwidth signals, which to a first approximation propagates as if these fluctuations are smoothed out.

5 Photonic Crystals and Four-Wave Mixing Phenomenon

This thesis contributes also to the four wave mixing phenomenon in photonic crystal waveguides. Four-wave mixing (FWM) is an important nonlinear phenomenon that may hold the key for many signal-processing applications in future optical networks, including wavelength conversion, signal regeneration, phase inversion, optical switching, and optical de-multiplexing. Degenerate FWM occurs when part of the optical power of a signal wave can be transferred to an idler wave located at another frequency through the mediation of a strong pump wave located at a third frequency. The efficiency of the energy exchange in this process is larger when the phases of the three waves are matched, i.e., when $\phi = 2\phi_p\phi_s\phi_i$ is small, where ϕ_p, ϕ_s, ϕ_i denote the total phase of the pump, signal, and idler waves, respectively. The most commonly adopted figure of merit that characterizes FWM is the conversion efficiency defined by,

$$\eta \equiv \frac{P_i(L)}{P_s(0)}, \tag{5}$$

where $P_s(0)$ is the incident signal power, and $P_i(L)$ is the idler power at the output of the waveguide of total length L. Nanophotonic slow-light structures such as photonic crystal waveguides offer the possibility of achieving sub-wavelength light confinement, while at the same time enhancing nonlinear effects such as

FWM. The estimation of η can play a crucial role in the design of the waveguide and guide us through the choice of several geometric, material, and signal parameters. In the case of degenerate FWM, the evolution of the three waves is generally described by a system of coupled ordinary differential equations. By solving this system of equations, one can in principle estimate η. Accounting for nonlinear losses complicates the problem, rendering the derivation of an exact analytical expression extremely difficult. In semiconductor materials such as silicon, nonlinear losses usually stem from two-photon absorption (TPA) and free-carrier (FC) generation. Self-phase modulation (SPM), cross-phase modulation (XPM), and dispersion should also be taken into account. Another complication arises from the fact that the wave parameters can exhibit substantial frequency dependence in PCWs, especially in the slow-light regime. Even if the waveguide is designed to ensure a smooth linear loss and group index frequency dependence, there is no guarantee that the nonlinear propagation parameters such as the effective modal areas A for all three waves will be the same. In fact, recent studies argue that SPM, XPM, and FWM may each perceive different values for A, unlike the case of a weakly guiding dielectric fiber, where such intricacies can be ignored. The results presented in this dissertation indicate that, the modal areas can exhibit strong frequency dependence even inside the flat-band region of the waveguide. It is therefore incorrect to assume the same modal area for all three waves, especially when the detuning is larger. Also the modal areas corresponding to each phenomenon may differ significantly in the case of large detuning. This thesis presents approximate analytical expressions for the FWM conversion efficiency η, when linear and nonlinear losses affect the propagation of the three waves. The usefulness of these formulas is twofold: first they provide significant insight into the nature of the FWM phenomenon from a theoretical point of view. They can also provide a target optimization function that requires much less computational time than the numerical solution of ODEs, when designing the PCW for nonlinear signal-processing applications. Unlike the design of PCWs for buffering applications, when optimizing the waveguide for FWM applications, one must also consider a multitude of signal parameters such as the pump-signal wavelength detuning and the incident pump power, which necessitate a large number of efficiency calculations for each structure. Simple analytical expressions can therefore speed up the optimization process. The approximate analytical expression of the FWM conversion efficiency η, assuming that only the nonlinear TPA loss is given by,

$$\eta = (\frac{\omega_i}{\omega_s})(1 + \frac{\kappa^2}{4g^2}) sinh^2(gL) e^{-\alpha_i L - 2ReT_i \overline{P_p} L}. \tag{6}$$

In the above equation, $\omega_\mu = 2\pi c/\lambda\mu$ where λ_μ is the wavelength for wave μ, κ is the total phase mismatch and g is the parametric gain. The parameter $T_i = (2jn_2\omega_i c^{-1} - \beta_{TPA}) S_p S_i A_{pii}^{-1}$ where n_2 is the nonlinear Kerr coefficient, β_{TPA} is the TPA coefficient, S_m is the slow-down factor of the m wave and A_{pii} is the effective modal area of the XPM. The average pump power is calculated by using the derived expression,

$$\overline{P_p} = \frac{A_{ppp}}{\beta_{TPA} S_p^2 L} ln(1 + \frac{\beta_{TPA} S_p^2 P_p(0)}{\alpha_p A_{ppp}}[1 - e^{-\alpha_p L}]) \tag{7}$$

TPA is typically accompanied by FC generation causing an additional absorption and dispersion. As discussed by several authors, FC effects can be significantly reduced when either a low repetition/low duration pulsed pump is used or when an external DC field is applied driving the FCs away from the center of the waveguide. We have compared the efficiency values obtained by the presented expression against the numerical solution of the coupled ODE equations, with respect to the wavelength of the signal and the idler waves. In this thesis we visually infer that overall the approximate formula provides an adequate description for medium- to high-efficiency values, which are important from a practical point of view. To quantify the error in the approximation, we calculated the average error e_5 and e_{10} between the numerical and the analytical efficiency (measured in dB) for wavelength combinations in which the ODEs efficiency is not lower than 5dB and 10dB compared to η_{max}, respectively. We obtain $e_5 = 0.35dB$ and $e_{10} = 1.1dB$, implying very good agreement for efficiency values of practical interest.

If no measures are taken, the FC generation can severely limit the FWM conversion efficiency. When FC effects are included, deriving an analytical approximation for η is much more involved. For one thing, the pump power cannot be obtained in exact form as in the previous cases. To that end, this thesis present two alternative methods for obtaining $P_p(z)$, which can be used in the estimation of η. First we may assume that the three loss types (FC absorption, TPA, and linear loss) act independently and that the overall pump loss can be approximated by the product of the three loss factors. This assumption leads to the following expression for the pump power,

$$P_p(z) = P_p(0) \frac{e^{-a_p z}}{(1 + K_1 z)(1 + K_2 z)^{1/2}} \tag{8}$$

where we have defined the parameters $K_1 = P_p(0)\beta_{TPA} S_p^2 / A_{ppp}$ and $K_2 = 4P_p(0)^2 Re\{F_p\}$ Adopting this first-order approximation for the exponential, we may readily obtain a closed-form formula for the average pump power:

$$\overline{P}_p = \left[\frac{2(e_1 - e_0 K_1)}{L K_1^{3/2} \sqrt{K_1 - K_2}} \tanh^{-1}\left(\frac{\sqrt{K_1}\sqrt{1 + K_2 z}}{\sqrt{K_1 - K_2}} \right) + \frac{2 e_1 \sqrt{1 + K_2 z}}{L K_1 K_2} \right]_0^L, \tag{9}$$

where $[f(z)]_b^d = f(d) - f(b)$. The coefficients e_0 and e_1 can be obtained so that the difference between the exponential and its first-order approximation is minimum in the least square sense inside $[0, L]$, in which case we find that,

$$e_0 = 2l_0^{-1}\left[\left(e^{-l_0} + 2\right) + 3l_0^{-1}\left(e^{-l_0} - 1\right)\right], \tag{10}$$

$$e_1 = -6l_0^{-1}L^{-1}\left[\left(e^{-l_0} + 1\right) + 2l_0^{-1}\left(e^{-l_0} - 1\right)\right], \tag{11}$$

with $l_0 = a_p L$ A second alternative is to solve the pump power in the case where the nonlinear loss is dominated by the FC absorption, i.e., $Re\left\{F_p\right\} P_p >> Re\left\{T_p\right\}$, in which case the pump power is given by the following equation,

$$P_p(z) = \frac{P_p(0)e^{-a_p z}}{\left(1 + \delta\left(1 - e^{-2a_p z}\right)\right)^{1/2}}, \tag{12}$$

where δ is given by $\delta = 2a_p^{-1} Re\left\{F_p\right\} P_p(0)^2$. Integrating with respect to z and dividing with the waveguide length, we readily obtain the following expressions for the average pump power and the average square pump power,

$$\overline{P}_p = -\frac{P_p(0)}{a_p L\sqrt{\delta}} \left\{\sin^{-1}\left(\frac{e^{-a_p L}}{\sqrt{\delta^{-1}+1}}\right) - \sin^{-1}\left(\frac{1}{\sqrt{\delta^{-1}+1}}\right)\right\} \tag{13}$$

Once the pump power is obtained by either one of the two methods discussed above, we can proceed to the estimation of the approximate efficiency. We arrive at the expression for η_0 given by,

$$\eta = l_i \left(\frac{\omega_i}{\omega_s}\right)\left(1 + \frac{\kappa_{tot}^2}{4g^2}\right)\sinh^2(gL), \tag{14}$$

where the total phase mismatch κ is replaced by κ_{tot}, which is determined by,

$$\kappa_{tot} = \kappa + Im\left\{F_s + F_i - 2F_p\right\}\overline{P}_p^2. \tag{15}$$

We validate the results obtained by the above analytical formula considering both aforementioned methods for estimating the pump power. We compare the FWM conversion efficiency obtained analytically against the numerical solution with respect to the wavelength of the signal and the idler waves. The presented results show that an overall good agreement is obtained between the numerical and the analytical solution for medium- to high-efficiency values. The average error for (λ_i, λ_s) combinations for which the ODE efficiency is not smaller than 5 dB than η_{max} is $\varepsilon_5 = 0.53dB$ and $\varepsilon_5 = 0.28dB$ calculating the pump power with Eq. 8 and Eq. 12, respectively. The same case for which the ODE efficiency is not smaller than 10dB than η_{max} is $\varepsilon_{10} = 2.03dB$ and $\varepsilon_[10 = 1.95dB$, respectively. The results obtained are calculated based on state-of-the-art fast-light linear loss levels and values of β_{TPA} corresponding to silicon. We note that the analytical formulas provide accurate results of the FWM conversion efficiency compared to numerical calculations. This thesis also briefly examines how the nonlinear loss due to FC generation and its impact on the efficiency η can be estimated in the case of a pulsed pump. The time evolution of the FC density, N_C is given by,

$$\frac{\partial N_C}{\partial t} = \frac{N_0 - N_C}{\tau_C} \tag{16}$$

, where $N_0 = \beta_{TPA}\tau_C S_p^3 P_p^2(z,t)/2\hbar\omega_p A_{ppp}^2$ is the FC density in the continuous-wave regime. We assume that the input pump signal has a period equal to T

and is comprised of rectangular pulses and duration equal to T_1. We can assume that dispersion effects do not significantly affect the pulse shape. Therefore the pump pulse will approximately retain its rectangular shape along the propagation length, and only its peak power will decrease because of loss. Solving the above equation for the n^{th} pulse period $[t_n, t_{n-1}]$ where $t_n = nT$, one finds that,

$$N_C(z,t) = N_C(z,t_n)e^{-(t-t_n)/\tau_C} + N_0 \left\{ e^{-t_n/\tau_C} - e^{-(t-t_n)/\tau_C} \right\} \tag{17}$$

during the on period of the pulse $t_n < tt_n + T_1$ and

$$N_C(z,t) = N_C(z,t_n+T_1)e^{-(t-t_n-T_1)/\tau_C} , \tag{18}$$

if $t_n + T_1 < tt_n + 1$. In the initial pulse periods (small n), there will be a gradual buildup of FCs until one reaches a point where the FC density $N_C(z,t_n)$ at the start of each period will be the same regardless of n. Under this condition, one obtains $N_C(z,t_{n+1}) = N_C(z,t_n)$, and combining the two last equations, we find that,

$$N_C(z,t_n) = N_0 \left\{ e^{-t_n/\tau_C} - e^{-T_1/\tau_C} \right\} \frac{e^{-(T-T_1)/\tau_C}}{1-e^{-T/\tau_C}} \tag{19}$$

We can easily calculate the average carrier density N_{avg} inside the pulse duration and use this carrier density in the estimations of the loss coefficient. Our results indicate that as the repetition rate becomes smaller, at some point T becomes much larger than $_C$, and the generated FCs have the necessary time to fully recombine before the next pulse arrives. Therefore in this regime, $N_C(z,t_n) \simeq 0$, and the nonlinear losses are due solely to the carriers generated inside the current pulse period, which do not depend on T and the repetition rate. As a consequence, the efficiency tends to remain constant at small repetition rates. For repetition rates above 1 GHz, an exponential degradation of η is observed. In this case, carriers generated in the previous pulse duration do not recombine fully, and there is a buildup of carriers, which increase the nonlinear loss.

6 Design Optimization

One of the main contributions of this thesis is the study of photonic crystal design with respect to the storage capacity. We define the storage capacity of the photonic crystal waveguides as the ratio of the achieved delay $L_W/|v_g|$ to the bit duration $1/R_b$, i.e.,

$$N_{\max} = L_W R_b / |v_g| , \tag{20}$$

where L_W is the waveguide length and v_g is the group velocity of the defect mode that carries the signal. N_{max} is therefore a function of R_b but also of the wave vector k and the geometry of the waveguide superlattice. Formally we can write $N_{max} = f(R_b, \alpha, k, r_\alpha, \varepsilon_\alpha, \varepsilon_b, h, x_1, y_1, r_1, , x_N, y_N, r_N)$, where N is the number of hole classes considered in the optimization, r_α is the radii of the lattice holes, α is the lattice constant, and ε_α and ε_b are the relative dielectric constants of the high- and low-index material, respectively. The function f is not known in closed form, but it can be computed using a plane wave expansion eigenmode solver to obtain the modal fields and the dispersion relation $k = k(\omega)$ of the waveguide. We apply standard optimization methods to choose the arguments of f in order to maximize N_{max}. To estimate N_{max}, one must estimate L_W, which is determined by the maximum tolerable optical loss and the dispersion-induced pulse broadening. In this thesis, we consider that the loss limit is $l_{max} = 20dB$, which can be easily compensated by semiconductor optical amplifiers. Given the optical loss coefficient $\varGamma$ of the waveguide in dB/cm, the maximum propagation distance due to losses is simply $L_\varGamma = l_{max}/\varGamma$. We also considered the maximum allowable length due to dispersion L_B is,

$$L_B = K\left(B_{\max}^2 - 1\right)^{\frac{1}{2}}\left(\beta_2^2 R_b^4 + \frac{1}{4}K^{-1}\beta_3^2 R_b^6\right)^{-\frac{1}{2}} \tag{21}$$

where, β_2 and β_3 are the GVD and third-order dispersion coefficients respectively, $B_{max} = 1.3$ is the maximum allowable broadening factor, and $K = 0.0224$. We choose the maximum propagation length L_W as the minimum of L_B, $L_\varGamma$, and L_{max} where $L_{max} = 1cm$ is the waveguide length limit imposed by optical integration considerations. In this thesis, we also discuss how the optimization procedure can be applied in order to design a PCSW delay line from scratch, considering the effect of multiple design parameters. We choose the standard W1 waveguide as a starting point with $r_\alpha = 0.27\alpha$, and perform a step-by-step optimization gradually increasing the number of parameters considered. Our presented results revealed a design with optimum storage capacity $N_{max} = 31.3bits$ at $n_g \simeq 24$ for $\varDelta y_1 = 0.1297\alpha$, $\varDelta y_2 = 0.0248\alpha$, $\varDelta y_3 = 0.0399\alpha$, and $\varDelta r_1 = 0.25\alpha$, considering $R_b = 40Gb/s$. In the same chapter, we also examine the case of $R_b = 100Gb/s$, where the maximum capacity obtained was $N_{max} \simeq 65bits$ at $n_g \simeq 21$ for $\varDelta y_1 = 0.14\alpha$, $\varDelta y_2 = 0.025\alpha$, $\varDelta y_3 = 0.018\alpha$, $\varDelta r_1 = 0.26\alpha$.

This dissertation contributes to the definition of new figure of merits concerning optimizing the photonic crystal waveguide design with respect to the FWM phenomenon. Simply achieving a large η is not always sufficient in many applications, since other important aspects need to be evaluated. For a given length L and pump power P_0, one should also be interested in the available bandwidth, which can be quantified in terms of the optical pump wavelength range $\varDelta\lambda$ in which does not fall below a certain level (say 3 dB) of its maximum value $\eta_0(P_0, L)$. Tunability is another important aspect that can be quantified as the wavelength separation $\delta\lambda$ between the pump and the signal waves for which η is again higher than 3dB compared to η_0. We define the product of the maximum efficiency, bandwidth, and tunability (EBT),

$$= \eta_0 \times \varDelta\lambda \times \delta\lambda \tag{22}$$

A large EBT value should ensure a smooth wavelength dependence for η, which is important in wavelength division multiplexing systems. Also, since modern trends in optical research dictate the use of compact, low-power components, we may also use a more powerful and size-aware FoM,

$$EBT_{PL} = \frac{\eta_0 \times \Delta\lambda \times \delta\lambda}{P_0 \times L} \tag{23}$$

Optimizing the waveguide with respect to EBT_{PL} is expected to yield shorter structures requiring less power at the expense of a smaller overall efficiency. We maximize the above FoMs based on an interior-point optimization method that combines a direct method for solving the constrained maximization problem, along with conjugate gradient steps using trust regions performed by MATLABs fmincon function. In our calculations, we assume a silicon PCSW with fixed $\alpha = 412nm$, $h = 0.5\alpha$ while the radii of holes not included in the optimization are also held fixed at $r_\alpha = 0.27\alpha$. We also assume that the range of the design parameters is $0.04\alpha\Delta r_i 0$, $0\Delta y_i 0.15\alpha$ and $0.1WP_0 2W$, $25\mu mL510\mu m$. The photonic crystal waveguide design obtained by maximizing EBT FoM is obtained for $\Delta y_1 = 0.149\alpha$, $\Delta y_2 = 0.099\alpha$, $\Delta y_3 = 0.012\alpha$, $Deltar_1 = 0.23\alpha$, $\Delta r_2 = 0.24\alpha$, $\Delta r_3 = 0.27\alpha$, yielding $EBT = 7.9nm^2$. Similarly, the optimum photonic crystal waveguide design with respect to the EBT_{PL} FoM is obtained for $\Delta y_1 = 0.151\alpha$, $\Delta y_2 = 0.11\alpha$, $\Delta y_3 = 0.013\alpha$, $\Delta r_1 = 0.23\alpha$, $\Delta r_2 = 0.27\alpha$, $\Delta r_3 = 0.27\alpha$, yielding $EBT_{PL} = 73.9fm/W$.

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